Suppression of T_1-type decoherence of phase qubits using uncollapsing and quantum error detection/correction

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Motivation:

It is fundamental to the realization of large scale quantum computations to suppress or correct the effects of decoherence. Here, we have investigated the performance of some schemes for minimizing the effects of energy relaxation for phase qubits in a quantum memory setup. As a measure of performance of these methods we have used the average state fidelity of a stored qubit state.

Energy Relaxation:

The quantum operation of energy relaxation can be represented as a probabilistic decay of the quantum state into the ground state by using a specific operator sum decomposition of the process. Assuming that each qubit (the logical and each ancilla) relaxes independently, this operator representation can be simply extended into the combined Hilbert space.

Single Qubit Relaxation

$$A_R = \begin{pmatrix} 0 & \sqrt{p_r(t)} \\ 0 & 0 \end{pmatrix} \qquad A_D = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p_r(t)} \end{pmatrix}$$

$$\rho(t) = A_D \rho(0) A_D^\dagger + A_R \rho(0) A_R^\dagger$$

$|\psi(t)\rangle \to |0\rangle$ with probability P_R

$|\psi(t)\rangle \to |\psi_D\rangle$ with probability P_D

$$|\psi_D\rangle = \frac{A_D|\psi\rangle}{\sqrt{\langle\psi|A_D^\dagger A_D|\psi\rangle}}$$

$$\rho(t) = P_R(t)|0\rangle\langle 0| + P_D(t)|\psi_D\rangle\langle\psi_D|$$

Multiple Qubit Relaxation

$$A_{DD}^{Q1Q2} = A_D \otimes A_D$$
$$A_{DR}^{Q1Q2} = A_D \otimes A_R$$
$$A_{RD}^{Q1Q2} = A_R \otimes A_D$$
$$A_{RR}^{Q1Q2} = A_R \otimes A_R$$

$|\psi\chi\rangle \to |\psi_D\chi_D\rangle$ with probability $P_D P_D$

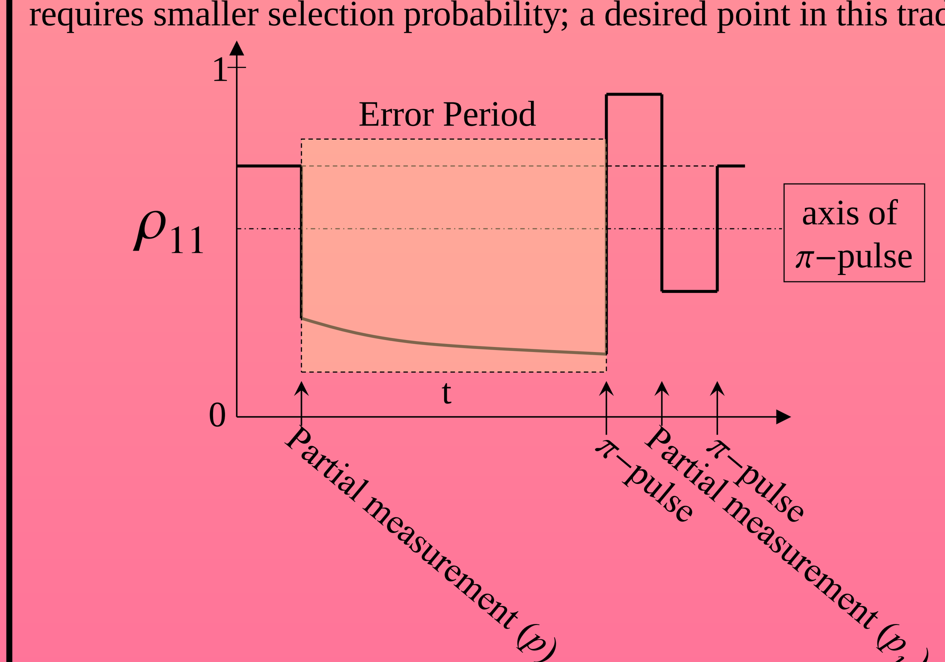
$|\psi\chi\rangle \to |\psi_D 0\rangle$ with probability $P_D P_R$

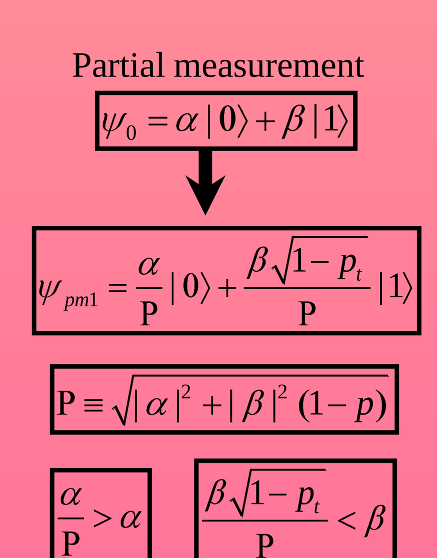
$|\psi\chi\rangle \to |0\chi_D\rangle$ with probability $P_R P_D$

$|\psi\chi\rangle \to |00\rangle$ with probability $P_R P_R$

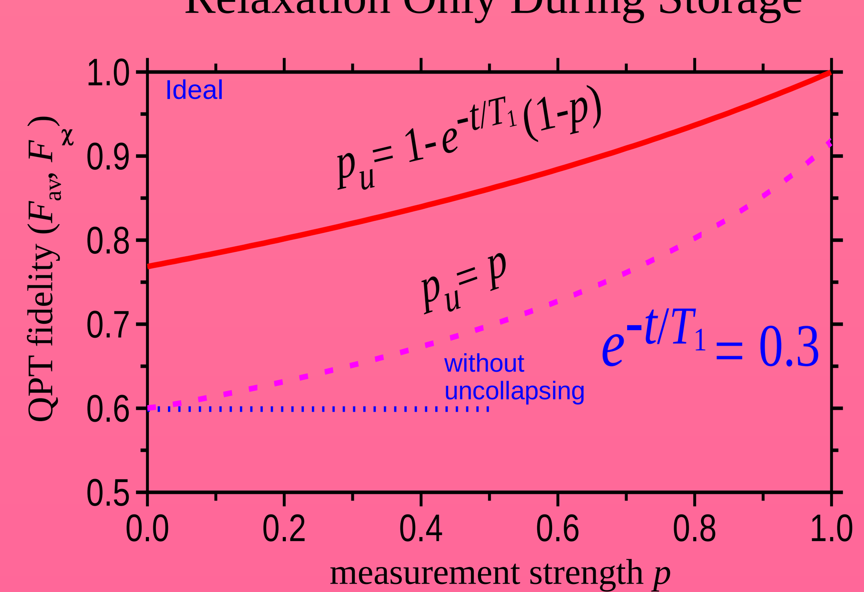
Uncollapsing:

Uncollapsing is a probabilistic reversal of a partial quantum measurement by a second partial measurement which yields an exactly contradictory result, thus providing no information about the state of the system. To protect the qubit state, the first partial quantum measurement moves it toward the ground state, where it is kept during the storage period, while the second partial measurement restores the initial state. This procedure preferentially selects the cases without energy decay events. Stronger decoherence suppression requires smaller selection probability; a desired point in this trade-off can be chosen by varying the measurement strength.

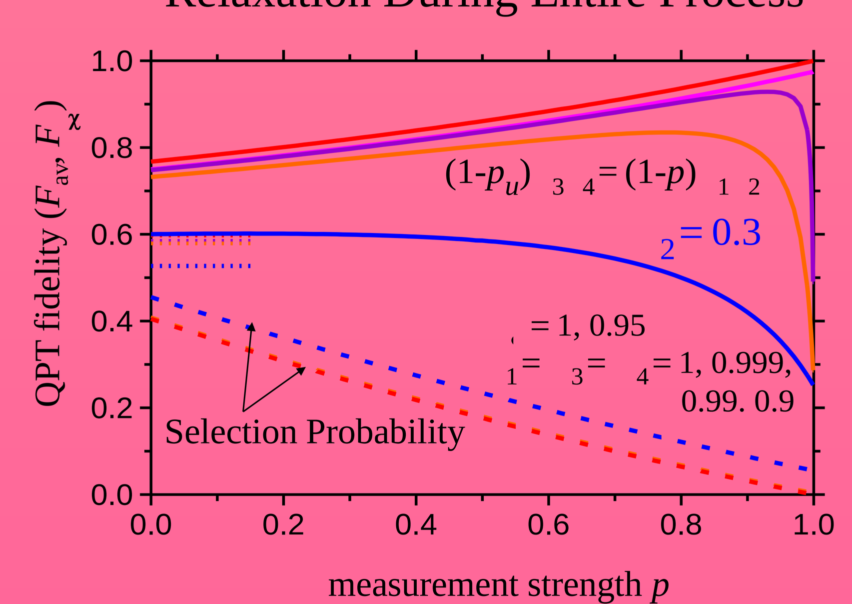




Relaxation Only During Storage

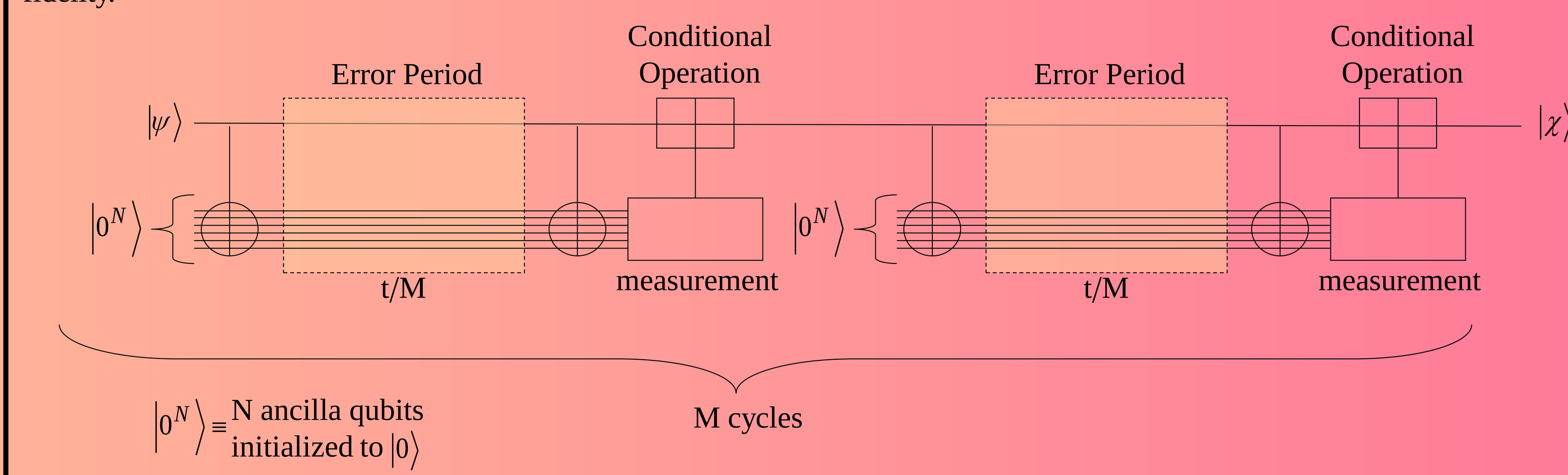


Relaxation During Entire Process

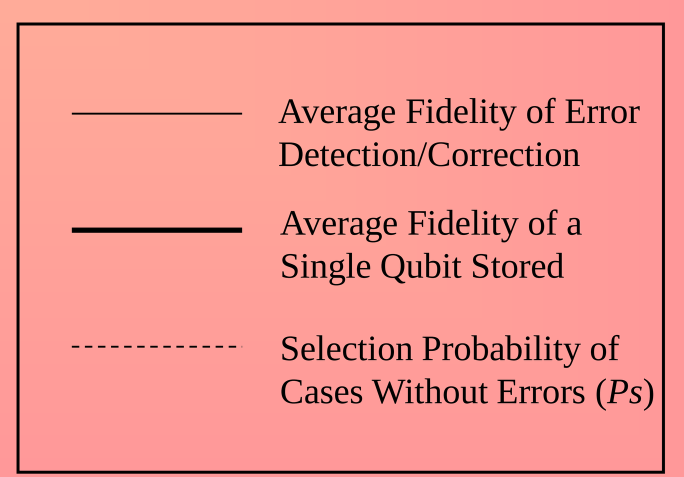


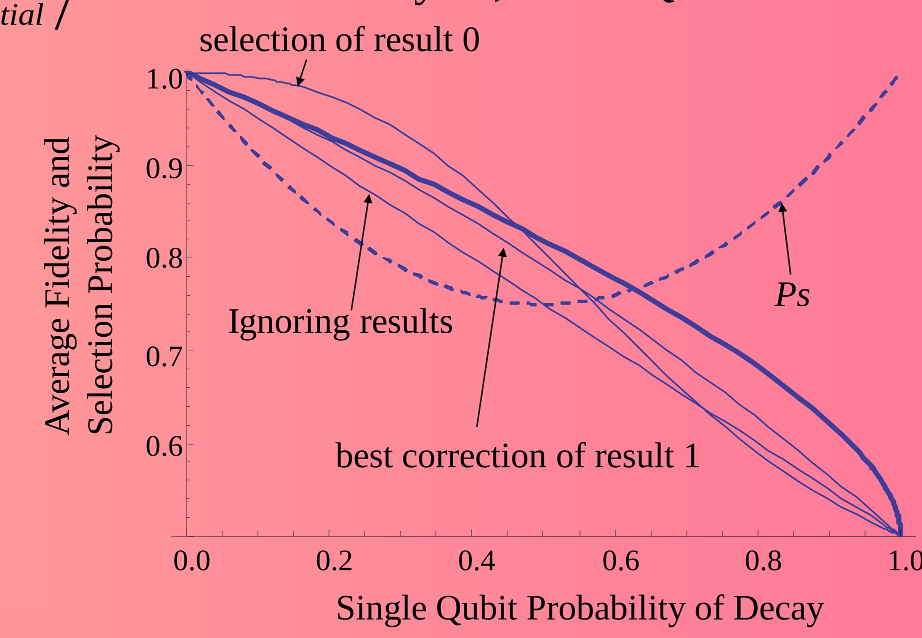
Compact error detection/correction codes:

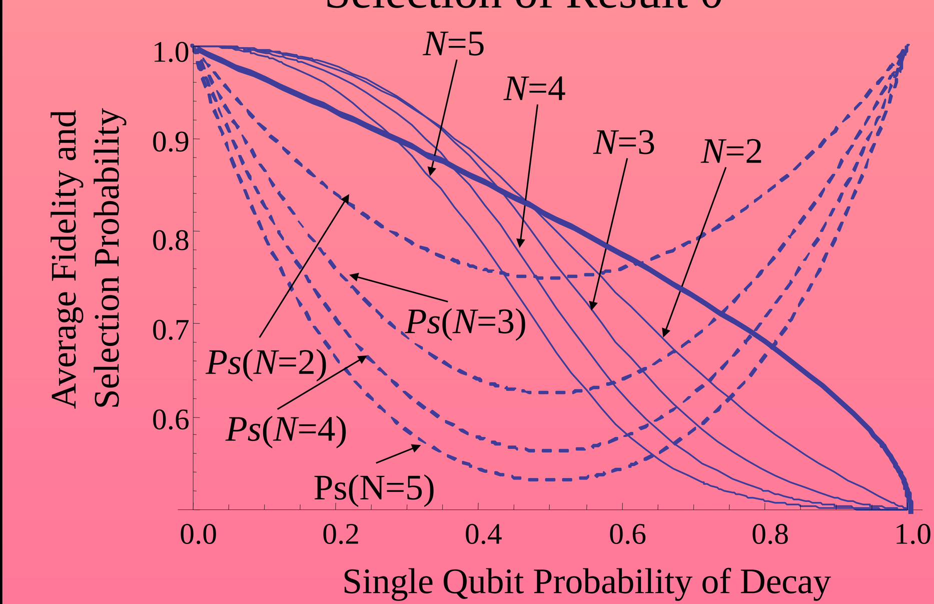
We consider a simple group of codes similar to the three-qubit bit-flip error correction code. However, we look at the performance of these codes in the presence of energy relaxation events rather than bit flip errors. It is also possible to use this group of codes as error detection protocols by selecting only the quantum states that result when certain measurement results occur. This is similar to uncollapsing in the sense that the information is preserved probabilistically with some degraded fidelity.

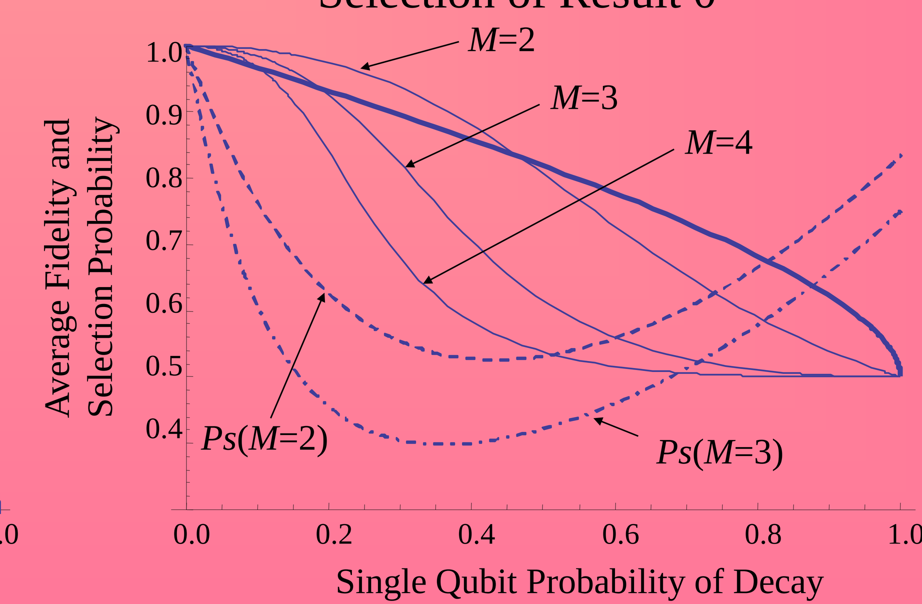


$$F_{av} \equiv \oint Tr\left(\rho_{final}\,|\psi_{initial}\rangle\langle\psi_{initial}|\right) d|\psi_{initial}\rangle$$









Conclusions

Both error detection/correction and uncollapsing schemes offer an increase in fidelity compared to an unprotected qubit

Both schemes cannot be efficiently cycled in case of errors due to energy relaxation

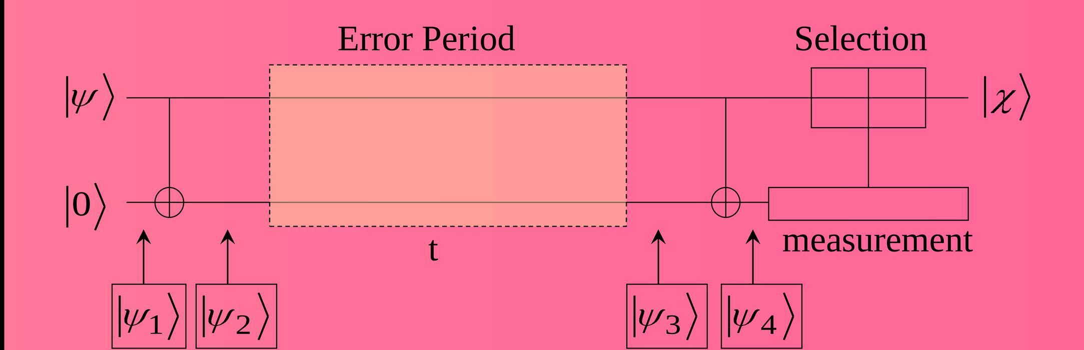
Analyzed detection/correction protocol works well for error detection, but is not good for error correction in the case of errors due to energy relaxation, even though it works well for bit-flip errors

Uncollapsing allows tuning of the selection probability by a parameter (measurement strength), while for the error detection codes the selection probability is determined by the storage period duration only

In order to have improvement compared to an unprotected qubit, the decay probability in the error detection/correction protocols should be below certain thresholds

These schemes may allow access to longer algorithms by extending available storage times

A simple example



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |\psi_1\rangle = \alpha|00\rangle + \beta|10\rangle \qquad |\psi_2\rangle = \alpha|00\rangle + \beta|11\rangle$$

$$|\psi_3\rangle = \begin{cases} \frac{\alpha|00\rangle + \beta(1-p)|11\rangle}{\sqrt{|\alpha|^2 + |\beta|^2(1-p)^2}}, & \text{with prob. } P_{nor} = |\alpha|^2 + |\beta|^2(1-p)^2 \\ |10\rangle, & \text{with probability } P_{10} = |\beta|^2(1-p)p \\ |01\rangle, & \text{with probability } P_{01} = |\beta|^2 p(1-p) \\ |00\rangle, & \text{with probability } P_{00} = |\beta|^2 p^2 \end{cases}$$

$$|\psi_4\rangle = \begin{cases} \frac{\alpha|0\rangle + \beta(1-p)|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2(1-p)^2}}|0\rangle, & \text{with prob. } P_{nor} = |\alpha|^2 + |\beta|^2(1-p)^2 \\ |1\rangle|1\rangle, & \text{with probability } P_{10} = |\beta|^2(1-p)p \\ |0\rangle|1\rangle, & \text{with probability } P_{01} = |\beta|^2 p(1-p) \\ |0\rangle|0\rangle, & \text{with probability } P_{00} = |\beta|^2 p^2 \end{cases}$$

After the measurement is taken, the state of the upper qubit depends on the measurement result of the lower qubit. The measurement result of the lower qubit is either 1 or 0. Keeping the upper qubit only for a certain measurement result (1 or 0) leaves the upper qubit in one of two mixed states, labeled by the corresponding measurement result: ρ_1 or ρ_0. There is also the possibility that we ignore or do not observe our measurement result, this will leave the upper qubit in the state ρ_{NOT}.

$$\rho_0 = \frac{P_\chi|\chi\rangle\langle\chi| + P_{00}|0\rangle\langle 0|}{P_\chi + P_{00}} \qquad \rho_1 = \frac{P_{10}|1\rangle\langle 1| + P_{01}|0\rangle\langle 0|}{P_{10} + P_{01}}$$

$$\rho_{NOT} = P_\chi|\chi\rangle\langle\chi| + P_{10}|1\rangle\langle 0| + P_{01}|0\rangle\langle 1| + P_{00}|0\rangle\langle 0|$$

If the measurement result is 1, a quantum error has been detected, in which case, one of the qubits has relaxed, but not both. Upon selection of cases with this result the state loses all information pertaining to the original superposition (note $|b|^2$ cancels in ρ_1). This is a general result no matter how many ancillas are used. If only cases with any ancilla in state 1 are selected, then all quantum information is lost.

$$\rho_1 = \frac{|1\rangle\langle 1| + |0\rangle\langle 0|}{2}$$

If the measurement result is 0, then most of the information about the original superposition is retained. This is a degraded information because of the leakage through the possibility of both qubits having relaxed. The state fidelity and average fidelity of ρ_1 can be calculated and reveals the following dependence on the length of the storage period (single qubit probability of decay). Also of relevance is the selection probability averaged over all initial states, this is the probability of not detecting an error, which is plotted on the same graph (dashed line).